TIM245 Lecture 18 (6/5/17)

Agenda

1) Project Phase IV and HW 3

2) Finish Lecture 17

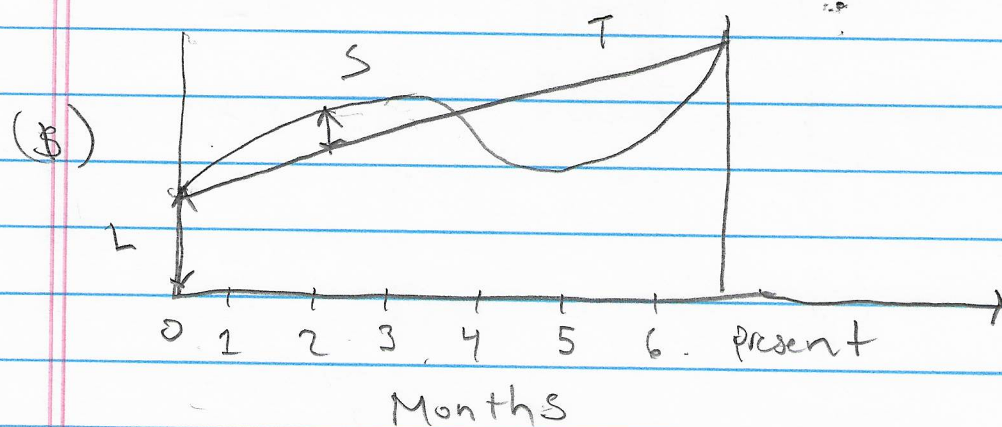
3) Time-Series Analysis Basic Concepts

4) Forecasting Models

③ Time Series Analysis Basic Concepts

Time series consists of a set of values observed over time.

Example: Revenue



observed values (D) = Systematic Component (S) + Random Component (R)

Systematic Component (S) = expected value

Level, L
"intercept" of the straight line approximation of D

Trend, T
Slope or rate of growth of D

Seasonality, S
Predictable Seasonal flucuations in D

Given historical observations

$(D_1, D_2, \ldots D_t)$

Two main objectives

<u>Decomposition</u>: Determine Level, Trend, and seasonality for D

<u>Forecasting</u>: Predict future observations

$(F_{t+1}, F_{t+2} \ldots F_{t+L})$ ← Length of the forecast

Comments on Forecasting

1) We can only forecast the Systematic Component, S, of the data.

2) The Random Component, R, can not be predicted.

3) Always plot the data before, during, and after when forecasting

(4) Forecasting Models

Two different techniques used in forecasting

Regression

Determining the Straightline that provides the best fit (e.g. least squares)

Smoothing

Determining the value of a particular obversation as a Combination of past observations (e.g. moving averages)

Two general approaches for applying regression and smoothing to forecast.

Static

Assume L, T, S do not vary with time

Adaptive

Update the estimations of L, T, S as data becomes available

Static Forecasting

Basic Idea: Use linear regression to forecast future values. Adjust these forecasts using a seasonal factor.

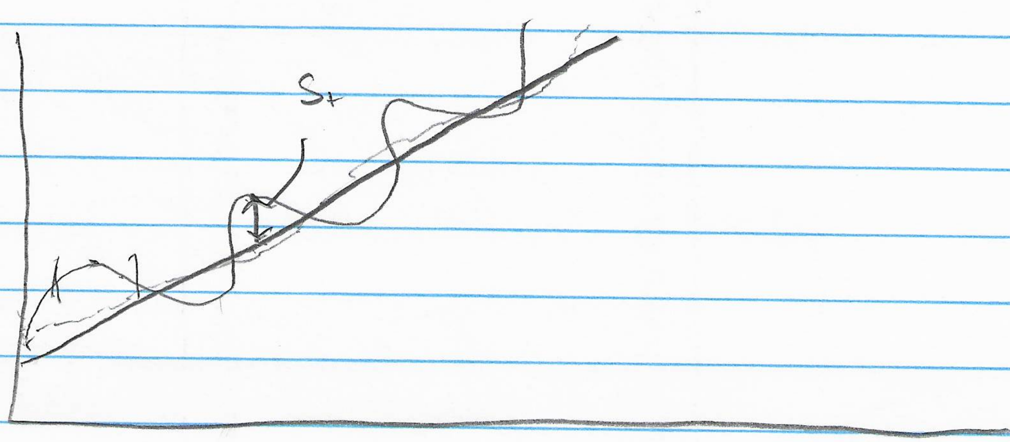
Process (Theory)

Step 1: Deseasonalize the given data

Deseasonalization is an averaging process where the average is taken over p points

$p \triangleq$ periodicity of the data, i.e. the number of points after which the seasonal pattern repeats.

Step 2: Regress the deseasonalized data



Step 3 Estimate the seasonal factor for each time period

$$\bar{S}_t = \frac{D_t}{\bar{D}_t}$$

D_t ← Actual Dat

$\bar{D}_t$ ← Regressed deseasonalized data

$(\bar{S}_1, \bar{S}_2 \ldots \bar{S}_t)$

Step 4 : Compute the average seasonal factor for each period

$(S_1, S_2, \ldots, S_p)$

Step 5 : Forecasting

$$\text{Seasonal Factor} = \frac{\text{Actual Data}}{\text{Regressed Deseasonalized data}}$$

Actual Data = (Seasonal factor)(regressed deseasonalized data)

$$F_t = (S_t)(\bar{D}_t)$$

$$F_t = (S_t)[L + (t)T] \quad (y = mx + b)$$

Adaptive Forecasting

Use each new observation to update either the level, L, or the trend, T, or the Seasonality, S, or any combination of L, T, S.

Three Cases

1) Level only: moving average, simple exponential smoothing

2) Level and Trend: Doble exponential smoothing (Holts method)

3) Level, Trend, and Seasonality: Triple exponential smoothing (Winter's method)

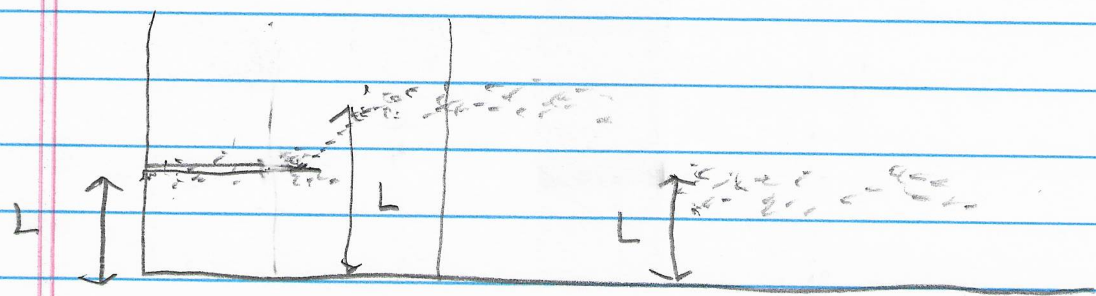
ARIMA can handle cases 1, 2, 3

$(p, d, q)(P, D, Q)$

Cases 1, 2 $\quad$ Case 3

Adaptive Forecasting: Moving Average

Assumption: Data has level, L, only



Step 1: Compute Level L_t

Select the number of data points, N, for computing the moving average

Example: 4 point moving average (N=4)

$$L_t = \frac{D_t + D_{t-1} + D_{t-2} + D_{t-3}}{4}$$

Step 2 : Forecast

$$F_t = L_{t-1}$$

Step 3 : Update (Adaptive)

Once the actual value $t+1$ is known we restimate the level

$$L_{t+1} = \frac{D_{t+1} + D_t + D_{t-1} + D_{t-2}}{4}$$

Repeat Step 2

Step 3

Simple Exponential Smoothing

Idea: Adjust our forecasts based on the error of our previous forecast.

Assumption: Data has Level, L, only

Step 1: Initialize Level

Compute $L_0 = \frac{1}{n}\left[\sum_{t=1}^{n} D_t\right]$

Step 2: Initial Forecast

$$F_1 = L_0$$

Step 3: Compute the forecast error

$E_t \triangleq$ error at time t

$$E_1 = F_1 - D_1$$

Step 4 Adjust the level based on the forecast error

$E_1 > 0 \Rightarrow F_1 > D_1 \Rightarrow$ over predicting

$E_1 < 0 \Rightarrow F_1 < D_1 \Rightarrow$ under predicting

We want to adjust L_1 based on the error E_1 to improve the forecasts.

$$L_1 = L_0 - \alpha E_1$$

$$D_1 = \text{Systematic } (S) + \text{Random } (R)$$

$$\Downarrow$$

$$E_1 \approx S + R \quad (\text{goal is to minimize } S)$$

α is a smoothing constant $(0 \leq \alpha \leq 1)$ that adjusts how much of the error is incorporated into the next forecast.

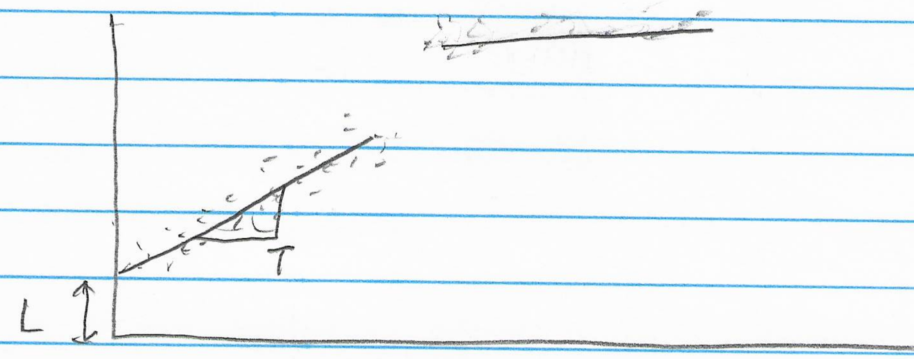
General Form:

Forecast $f_t = L_{t-1}$

Level $L_t = \alpha D_t + (1-\alpha) L_{t-1}$

Holts Method: Double Exponential Smoothing

Assumption: Data has Level, L, and trend, T, but no Seasonality



<u>Idea</u>: Regress the data to capture the initial Level, L_0, and the initial trend, T_0.

Use the values to forecast F_1

$$F_1 = L_0 + T_0(t)$$

Use two smoothing constants, α and β, to adjust the level and trend estimates based on the error.

General Form

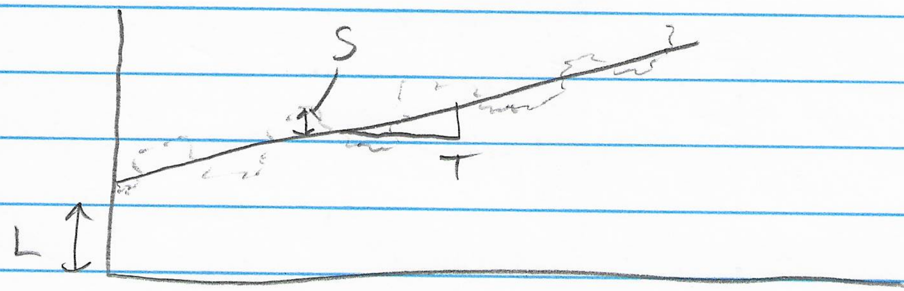
Forecast : $F_t = L_{t-1} + T_{t-1}$

Level : $L_t = \alpha D_t + (1-\alpha)(L_{t-1} + T_{t-1})$

Trend : $T_t = \beta(L_t - L_{t-1}) + (1-\beta) T_{t-1}$

Winter's Method: Triple Exponential Smoothing

Assumption: Data has Level, Trend, and Seasonality (L, T, S)



Idea: Initial Level, Trend, and Seasonality are obtained from the static method

$$F_1 = (L_0 + T_0) S_1$$

Use three smoothing constants to update our estimates for L, T, S

Forecast $= (L_{t-1} + T_{t-1}) S_{t-p}$

Level: $L_t = \alpha\left(\frac{D_t}{S_{t-p}}\right) + (1-\alpha)(L_{t-1} + T_{t-1})$

Trend: $T_t = \beta(L_t - L_{t-1}) + (1-\beta) T_{t-1}$

Seasonality: $S_t = \gamma \frac{D_t}{L_t} + (1-\gamma) S_{t-p}$